Pricing European Call Option and Sobolev Space Energy Estimate Theorem on Access Bank Share Prices

[1]Azor, P. A. and [2]Amadi, I. U.
[1] Department of Mathematics & Statistics,
Federal University, Otuoke, Nigeria,
Email: azorpa@fuotuoke.edu.ng
[2]Department of Mathematics & Statistics,
Captain Elechi Amadi Polytechnics, Port Harcourt,
Nigeria, email: innocent.amadi@portharcourtpoly.edu.ng

DOI: 10.56201/ijcsmt.v10.no2.2024.pg53.61

Abstract

This paper, examined Black-Scholes (BS) analytic approach for Call Option and Black-Scholes Partial Differential Equation (PDE) by means of Sobolev space Energy Estimate theorem to satisfactorily optimize good estimate of asset prices on the analysis of Access Bank share prices. The precise conditions which gave explicit prices of all forms of trading days disparities were completely obtained both in BS analytic formula and BS PDE through Sobolev space Energy Estimate theorem correspondingly. From the analysis of results shows the following: a little increase in the time of maturity expressively increases the value of assets ; increase in strike price likewise increases the value of assets ; the estimates of BS Call option prices outclassed Sobolev space energy estimates; the two averages of Call option prices and Sobolev space Energy estimate theorem to identified trends of share price of Access bank PLC for proper investment decisions; To this end, the surface-view graphical representations for both estimates were well discussed for the purpose of investment plans as it affects Access bank and capital market .

Keywords: *Access bank Share prices, Black-Scholes , BS PDE , Call Option and Sobolev spaces Energy Estimates*

INTRODUCTION

The extensive presentation of the modelling of financial quantities and variables in the field of science and technology has attracted numerous commendations. The modelling of financial quantities and variables, has a wide range of applicability which consist of valuation of financial derivatives which builds up the financial power of each investment and intensifies proficiency for Option traders, the stock volatility enables investors to estimate the up and down movement of stock prices, security of high-risk investments, and so on. Some assessments or evaluations in the daily activities in the financial markets, regarding time to maturity are a requirement for the Option pricing. Some researchers, such as [1] – [4], have carried out extensive work on the financial markets and obtained fascinating results. The comprehension of financial variables and its dynamic relationships with respect to its effects on investors is of paramount significance. Hence, it is necessary to have a comprehension of the dynamic nature of the physical problem to be solved, and also know the reason such method is appropriate. The analytical method, which can give exact solutions, are required to solving physical problem of this nature, which aids proper mathematical predictions. Basically, it is necessary to study financial markets and its related problems, using well formulated and accurate analytical solutions, so as to invigorate real-life results; hence, the analytical approach is adopted based on the Black-Scholes model for European Call Options.

Great interest in Financiers, Mathematicians and Statisticians has emerged over the Black-Scholes (BS) model, which was discovered by [5], for the analysis of the European Option on the stock market (which does not pay a dividend during the Option's life) in addition to solving PDEs in Sobolev spaces.

For example, [6] carried out studies on implied volatility and implied risk-free return rates in solving systems of Black-Scholes equations, in which they proved that Option prices give significant information for market players, for future expectations and market policies.

In like manner, [7] examined the Black-Scholes formula for assessment of European Options, where the Hermite Polynomials were utilized and deduced that the Black-Scholes formula can easily be achieved, deficient of the use of Partial differential equation. Furthermore, [8] studied the Black-Scholes terminal value problem and declared that their proposed approach was better and simpler than the previous approach. [9] carried out a study and incorporated time - varying factor in the explicit formula for various aspects of Options with the motive of providing exact solution for the payment of dividend and equity Option. [10] – [15] gives details of work on more financial models.

However, a lot of scholars have written extensively on Sobolev spaces with different dimensions and results obtained in diverse ways such as [16-19] and [20-22] etc.

On that note, this paper studied Black-Scholes (BS) analytic approach for Call option and Black-Scholes Partial Differential Equation (PDE) by means of Sobolev space Energy Estimate theorem to adequately enhance worthy estimate of asset prices on the analysis of Access Bank share prices. The precise conditions which gave explicit prices of all forms of trading days disparities were completely obtained both in BS analytic formula and BS PDE through Sobolev space Energy Estimate theorem consistently. Henceforth, this paper compliments the work of [19] as this will widen the area of applicability in this dynamic area of mathematics of finance.

The paper is set to follow: Section 2.1 Mathematical framework, Results and discussion is seen in 3.1. This paper is concluded in Section 4.1.

2.1 Mathematical Framework of Black-Scholes Model

The Black-Scholes model is made up of on seven assumptions:

the asset price has characteristics of a Brownian motion with μ and σ as constants; the transaction costs or taxes are not allowed; the entire securities are absolutely divisible; dividend is not permitted during the period of the derivatives; unacceptable of riskless arbitrage opportunities; the security trading is constant and the option is exercised at the time of expiry for both call and put options.

Here, we consider a market where the underlying asset price v $, 0 \le t \le T$ on a complete probability space $(\Omega, f, \wp)$ is governed by the following stochastic differential equation:

$$dS(t) = \alpha S(t)dt + \sigma dw(t).\ 0 < v < \infty\ . \tag{1.1}$$

Theorem 1.1: (Ito's formula) Let $(\Omega, \beta, \alpha, \mathrm{F}(\beta))$ be a filtered probability space $X = \{X, t \ge 0\}$ be an adaptive stochastic process on $(\Omega, \beta, \alpha, \mathrm{F}(\beta))$ possessing a quadratic variation (X) with SDE defined as:

$$dX(t) = g(t, X(t))dt + f(t, X(t))dW(t)$$

$t \in \Re$ and for $u = u(t, X(t) \in C^{1\times 2}(\Pi \times \square))$

$$du(t, X(t)) = \left\{\frac{\partial u}{\partial t} + g\frac{\partial u}{\partial x} + \frac{1}{2}f^2\frac{\partial^2 u}{\partial x^2}\right\}d\tau + f\frac{\partial u}{\partial x}dW(t)$$

Using theorem 1.1 and equation (1.1) comfortably solves the SDE with a solution given below:

$$S(t) = S_0\ \exp\left\{\sigma dW(t) + \left(\alpha - \frac{1}{2}\sigma^2\right)t\right\}, \forall t \in [0,1]$$

In mathematical finance, arbitrage arguments show that any derivative $V(\mathrm{S}, t)$ written on v must satisfy the partial differential equation of the form of option pricing; hence, we have the following:

$$\frac{\partial V(\mathrm{S}, t)}{\partial t} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 V(\mathrm{S}, t)}{\partial S^2} + rS\frac{\partial V(\mathrm{S}, t)}{\partial S} - rV(\mathrm{S}, t) = 0\ . \tag{1.2}$$

Where, r represents interest rate, σ represents volatility of the underlying assets and t represents time of maturity.

With boundary conditions:

$$V(\mathrm{S}, t) \to \infty\ as\ S \to \infty\ on\ [0, T).\ . \tag{1.3}$$

$$V(\mathrm{S}, t) \to 0\ \ as\ \ S \to 0\ on[0, T).\ . \tag{1.4}$$

And final time condition given by:

$$V(S_T, T) = (S_T - k)^+ = f(S_T)\ on\ [0, \infty]. \tag{1.5}$$

Equation (1.4) is the value of asset is worthless when the stock price is zero, [19]. The details of the above option model can be expressly be found in the following books: [17-22] etc.

To eliminate the price process in (1.2) slightly gives the Black-Scholes analytic formula for the prices of European call option is given as follows

$$\left.\begin{aligned} C &= SN\left(d_1\right) - Ke^{-rt}N\left(d_2\right) \\ d_1 &= \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \\ d_2 &= d_1 - \sigma\sqrt{T} \end{aligned}\right\} \tag{1.6}$$

Where, C is Price of a call option, S is price of underlying asset, K is the strike price, r is the riskless rate , T is time to maturity, σ^2 is variance of underlying asset, σ is standard deviation of the (generally referred to as volatility) underlying asset, and N is the cumulative normal distribution.

However, this study will effectively and to adequately analyse Access bank according to [15] in order to realistically assess the value of share prices

2.1.2 Empirical Illustrations of Sobolev Space Energy Estimate Theorem

Theorem 2.2. (Energy estimates). There is a constant C, which due depends majorly only on ϕ, T and their coefficients is of L ,such that

$$\begin{aligned} &\max_{0\le t\le T} \| \mathrm{u}_m(t) \| L^2(\phi) + \| \mathrm{u}_m \| \mathrm{L}^2\left(0,T;H_0'(\phi)\right) + \| u_m' \| L^2\left(0,T,H^{-1}(\phi)\right) \\ &\le C\left(\| f \| L^2(0,T;L^2(\phi)\right) + \| g \| L^2(\phi)), \end{aligned} \tag{1.7}$$

For $m = 1, 2, ...,.$ the proof of this theorem can be seen in works of [16-22] etc.

Here, we present some empirical illustration of Sobolev space energy estimate theorem and analyzed based on financial market variables using Matlab programming software. This is enables us observe the behaviour of some stock variables or quantities on value of asset at different maturity days worth of the investments. Therefore, the Energy estimate is assumed to be asset value function where the left-hand side of the estimate where constrained and the right-hand side were used as a function $u\left(t\right)$ which gave the following: Hence we have the following:

$$u\left(t\right) = C\left(\| f \| L^2(0,T;L^2(\phi)\right) + \| g \| L^2(\phi)).$$

Table 1: The value of Share price of Access Bank, PLC with variations of maturity days and trading average with the following parameter values: $r = 0.03,\ \sigma = 0.25,\ k = 450$.

initial share price $\left(S_0\right)$	call option prices when time $t = 6$	call option prices when time $t = 12$	**Average**
410 80	97.1219 0.0812	136.6974 1.2099	116.9096 0.6455

126	1.0494	5.9757	3.5126
79	0.0752	1.1537	0.6144
98	0.2706	2.5420	1.4063
92	0.1880	2.0277	1.1079
127	1.0925	6.1316	3.6120
91	0.1764	1.9490	1.0627
378	77.5702	114.9730	96.2716

Tables 1 and 3 show increase in the trading days increases the value of assets.

The implication of this is that more trading days gives the option holder more flexibility in terms of when to exercise the option or sell it. Potentially increasing its value. Increased trading days can also lead to greater volatility in the underlying stock, which can increase the value of call options. This remark is encouraging in every investment because it is profit maximizing which will guide the management of Access bank, PLC, the ways of taking decisions based on the levels of their investments.

Secondly, the two averages of the same Tables define the characteristics of an investment portfolio. An investor or investors can measure how well their investments have gone and compare them with other investment chances or market standards.

Thirdly the time disparities seen in Tables 1and 3 shows that Black-Scholes estimates are lesser than the application of Sobolev space energy estimate theorem. Therefore Black-Scholes analytic estimates are better.

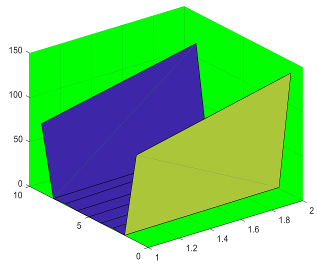

Figure 1: The Surface view representation of Black-Scholes Call option prices with variations of trading days

Figure 1 portrays continuous trading and accumulations of wealth for Access bank which is beneficial for capital investments. The trend movement is smooth but not steady due to volatility changes during the trading days.

Table 2: The Application of Energy Estimate Theorem of Sobolev spaces with variations of Strike prices when Share price of Access bank is 410.

Strike price (K)	Volatility (σ)	Time (T)	Value of the Stock $U(t)$
5	1.0000	2.0000	820.004
10	1.0000	2.0000	1640.004
15	1.0000	2.0000	2460.004

20	1.0000	2.0000	3280.004
25	1.0000	2.0000	4100.004
30	1.0000	2.0000	4920.004
35	1.0000	2.0000	574.004
40	1.0000	2.0000	6560.22

This is an estimates using Sobolev space energy estimate theorem which describes: An increase in strike price when both volatility and time are constants shows an increase in the value of assets. This means that the value of the option is influenced by current market price of the underlying assets and other significant factors. This is informs every Access bank management on vital ways of taking decision.

Table 3: The value of Share price of Access Bank, PLC with variations of Initial share prices and trading average with the following parameter values: $r = 0.2,\ \sigma = 0.03,\ K = 25$ and $g = 0.02$ **Applications of Energy Estimate theorem of Sobolev spaces.**

Initial share price (S_0)	Energy Estimate prices when time $t = 6$	Energy Estimate prices when time time $t = 12$	**Average**
410	123.0000	738.0001	430.5000
80	72.0001	144.0001	108.0001
126	113.4001	226.8001	170.1001
79	71.1001	142.2001	106.6501
98	88.2001	176.4001	132.3001
92	82.8001	165.6001	124.2001
127	114.3001	228.6001	171.4501
91	81.9001	163.8001	122.8501
378	340.2001	680.4001	510.3001

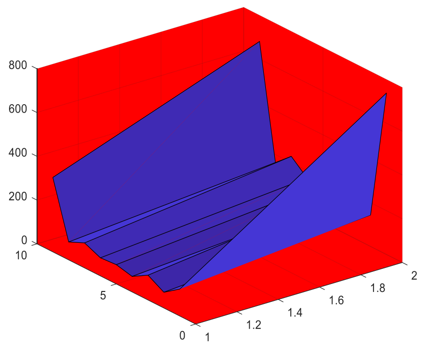

Figure 2: The Surface view representation of Sobolev space Energy Estimate theorem through Black-Scholes PDE with variations of trading days

Figure 2 depicts constants trading and progressions of capital for Access bank which is advantageous for capital investments plans. The trend movement is not smooth but not also steady due to volatility changes during the trading days and concepts of weak derivatives from its domain.

4.1 Conclusion

Estimates of share price investments are keys of proper management decisions. On that note, this paper examined Black-Scholes (BS) analytic approach for Call Option and Black-Scholes Partial Differential Equation (PDE) by means of Sobolev space Energy Estimate theorem to adequately optimize worthy estimate of asset prices on the analysis of Access Bank share prices. The precise conditions which gave explicit prices of all forms of trading days disparities were completely obtained both in BS analytic formula and BS PDE through Sobolev space Energy Estimate theorem consistently. From the analysis of results it was discovered that: a little increase in the time of maturity expressively increases the value of assets ; increase in strike price likewise increases the value of assets; the estimates of BS Call option prices outclassed Sobolev space energy estimates; the two averages of Call option prices and Sobolev space Energy estimate theorem to identified trends of share price of Access bank PLC for proper investment decisions; and the surface-view graphical representations for both estimates were significantly used to identify trends of asset variations in respect to timing.

However, investigating two or more banking share prices will be an interesting study to explore.

References

[1] M. S. Umar and B. M. Tijani, Stock Prices and Firm Earning per Share in Nigeria. Journal of

Research in National Development. 11 (2), 21 – 33 (2013).

[2] F. N. Nwobi, Application of Symmetry Analysis of Partial Differential Equations Arising from

Mathematics of Finance. Doctor of Philosophy in school of Mathematical Sciences University of

KwaZulu-Natal Durban South Africa, 2011.

[3] B. O. OSU, A Stochastic Model of The Variation of The Capital Market Price. International Journal

of Trade, Economics and Finance 1(2), 297 (2010), IACSIT Press.

[4] Davies, Iyai, Uchenna Amadi Innocent, Roseline Ndu, and others. Stability Analysis of Stochastic

Model for Stock Market Prices. International Journal of Mathematical and Computational

Methods 4 (2019), International Association of Research and Science.

[5] F. Black and M. Scholes, The Pricing of Options and Corporate Liabilities. Journal of Political Economy 81(3), 637-654(1973). The University of Chicago Press.

[6] R. Cont, Model Uncertainty and its Impact on the Pricing of Derivative Instruments. Mathematical Finance, 16 (3), 519 – 547 (2006), Wiley Online Library.

[7] E. Lindstrom, J. Strojby, M. Broden, M. Wiktorsson and J. Holst, Sequential Calibration of Options. Computational Statistics & Data Analysis 52(6), 2877-2891(2008), Elsevier.

[8] F. Lorella, F. Mariani, M. C. Recchioni, F. Zirilli and others, Calibration of a Multiscale Stochastic Volatility Model using Eurpean Option Prices, Mathematical Methods on Economics and Finance 3(1),49-61 (2008).

[9] B. Marcelo, M. Scott and S. Marco, Implied Volatility and the Risk-Free Rate of Return in Option Markets [Preliminary Draft]. Comments Welcome (2014).

[10] O. L. Babasola, I Irakoze and A.A. Onoja, Valuation of European options within the Black-Scholes Framework using the Hermite Polynomial. J. Sci. Eng. Res 5(2), 200-13 (2008).

[11] B. Shin and K. Hwajoon, The Solution of Black-Scholes Terminal Value Problem by Means of Laplace transform. Glob. J. Pure Appl. Math. 12, 4153 - 8 (2016).

[12] M. R. Rodrigo and R. S Mamon, An Alternative Approach to Solving the Black-Scholes Equation With Time-Varying Parameters. Applied Mathematics Letters 19(4), 398 - 402 (2006), Elsevier.

[13] B.O. Osu and C. Olunkwa, Weak Solution of Black-Scholes Equation Option Pricing with Transaction Costs. Int. J. Appl. Math. 1 (1), 43 - 55 (2014).

[14] F. N. Nwobi, M. N. Annorzie and I. U. Amadi, Crank-Nicolson Finite Difference Method in Valuation Of Options. Commun. Math. Fin, 8 (1), 93 - 122 (2019).

[15] B. O. Osu, S. C. Emenyonu, C. P. Ogbogbo and C. Olunkwa, Markov Models on Share Price Movements in Nigeria Stock Market Capitalization. Applied Mathematics and Information Sciences an International Journal N 2(2019).

[16] Amadi, I. U., Azor, P. A. and Chims, B. E. (2020). Crank-Nicolson Analysis of Black-Scholes Partial Differential Equation for Stock Market Prices. *Academia Arena*.12(1):1-22.

[17] Amadi, I.U.,Osu. B.O. and Davies, I.(2022). A Solution to Linear Black-Scholes Second order Parabolic Eqaution in Sobolev Spaces. *International Journal of Mathematics and Computer Research*,10, issue 10,2938-2946.

[18] Amadi, I.U., Davies, I. Osu , B. O. and Essi, I.D. (2022).Weak Estimation of Asset Value Function of Boundary Value Problem Arising in Financial Market, *Asian Journal of Pure and Applied Mathematics,* 4(3),600-615.

[19] Amadi, I. U., Onyeka, P. and Azor, P. A. (2024). Crank-Nicolson Finite Difference Method with Sobolev Space Energy Estimate Theorem for Capital Market Prices. *International Journal of Mathematics,* vol.7, issue 3,122-136.

[20] Osu, B. O and Amadi, I U. (2022). Existence of Weak Solution with Some Stochastic Hyperbolic Partial Differential Equation. *International Journal of Mathematical Analysis and Modeling*, 5, issue 2,13-23.

[21] Osu, B. O, Amadi, I. U. and Davies, I. (2022). An Approximation of Linear Evolution Equation with Stochastic Partial Derivative in Sobolev Space. *Asian Journal of Pure And Applied Mathematics*, *International Journal of Mathematical Analysis and Modeling,* 5, issue 3,13-23

[22] B.O. Osu and C. Olunkwa, (2014). Weak Solution of Black Scholes Equation Option Pricing with Transaction Costs. *International Journal of Applied Mathematics*. 1, 43.